Computational Modelling in the Physics Curriculum: an Example with the Wilberforce pendulum

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Abstract. We use a Wilberforce pendulum with a special MEMS sensor to present an interesting example of Computational Modelling employing either the free App Phyphox for smartphones/tablets plus the web-based modelling tool InsightMaker, or the integrated Coach7 system for both data-taking and modelling, to show how a detailed analysis of the motion and of the energetical aspects may be easily performed.

1. Experimental setup

The Wilberforce pendulum [1] is a didactical device often used in class demonstration to show coupled oscillations (rotational and longitudinal) producing beats in a special mass-spring set up. Our pendulum [2] uses an helicoidal spring and a brass cylindrical bob with 4 screws for fine-tuning of the inertial moment, and a MEMS absolute orientation sensor (including accelerometer and gyroscope) driven by an ESP32 microcontroller with Bluetooth and Li-ion battery charger. The top end of the spring is fixed to a brass cylinder that may both rotate along the vertical axis and shift in vertical direction. By a proper choice of initial displacement z_0 and rotation angle α_0 we may excite each one of the three different pendulum motions we are interested in.

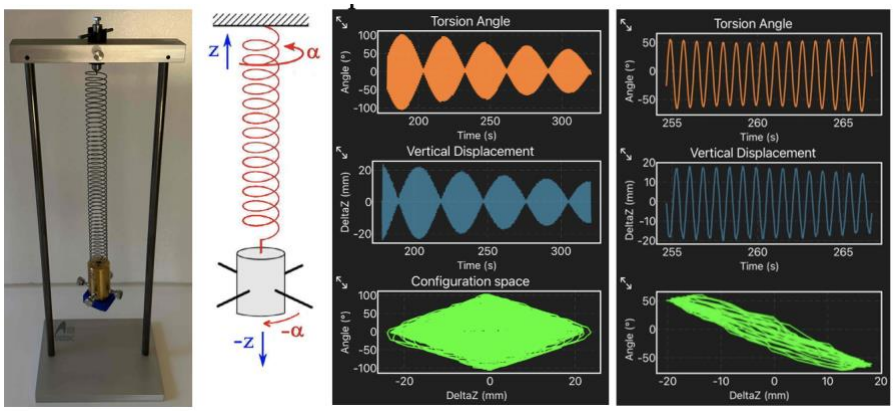

Figure 1. The Wilberforce pendulum and two examples of experimental results.

The complete equation for the Wilberforce pendulum energy may be written as sum of *Rotational* E_R, *Translational* E_T and *Coupling* E_e terms:

$E_{Wilber}=E_T+E_R+E_e=(mv^2+kz^2+I\omega^2+\delta\alpha^2+\varepsilon\alpha z)/2$ (1)

The parameters m, k, I and δ may be measured directly or calculated from measured motion.

The Normal Modes (no beatings) can be excited by choosing *initial conditions*: $z_0 = \pm\Gamma\alpha_0$, where $\Gamma = (I/m)^{1/2}$ is the *radius of gyration* of the pendulum.
The frequency difference in the two Modes $\omega_1 - \omega_2$ is the *beating frequency* $\omega_B = \varepsilon/2(kI)^{1/2}$.

2. Dynamical modelling

Both in *InsightMaker* [3] and *Coach 7* [4] we create the model using: **a**) "*reservoirs*" which represents physical quantities (extensive quantities to be "stored" in the system, as momentum $p=mv$, angular momentum $L\omega=I$; **b**) "*flow rates*", which implement, at each instant, the value of the incoming and/or outgoing exchanges of the reservoir to which they are connected; **c**) "*constants* or variables", which re present ge neral phy sical quantities ; d) "links", to conn ect the blocks.

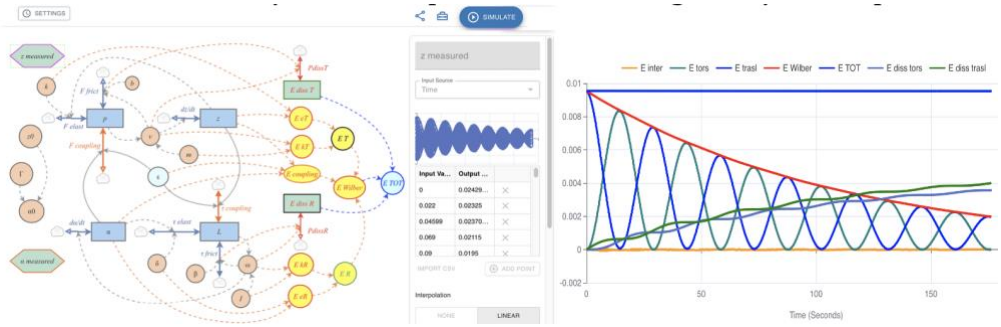

Figure 2. InsightMaker Dynamical model and some simulated energy-components.

In InsightMaker we need to import data recorded by Phyphox as a CSV file for a comparison between model results and experimental data.

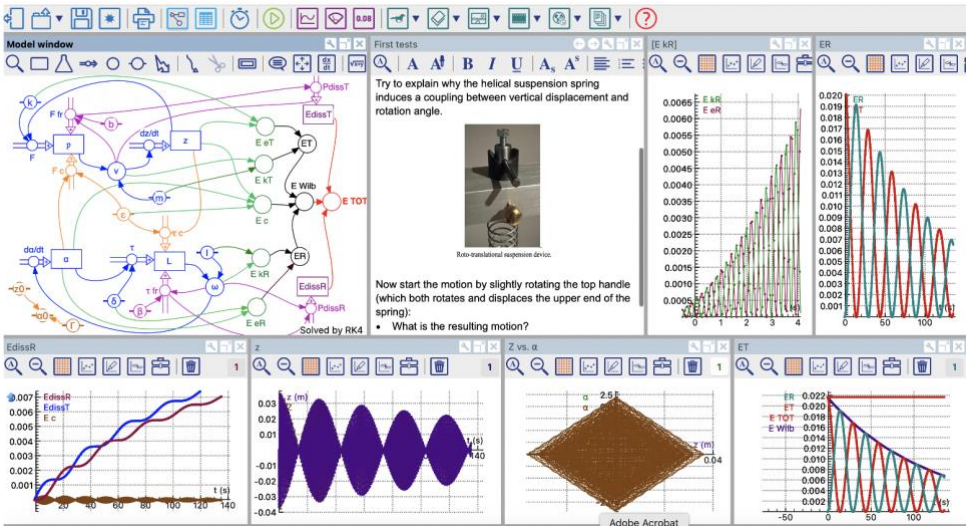

Figure 3. Coach7 model and the simulated energy-components

In Coach7 the data are first recorded with a *Measurrement Activity* and then imported into a *Modeling Activity* for comparison.

The model shows to the student that the energy is cyclically exchanged between translation and rotation oscillations, that in both oscillations the energy is converted from *elastic-potential* and *kinetic* terms, that *interaction energy* oscillates with frequency 2ω, with a beating frequency $2\omega_B$, and reaches zero amplitude every time the motion is purely rotational or purely translational.

All these aspects of the system-dynamics cannot be easily studied without modelling.

3. Conclusions

Here we described a novel apparatus that allows to independently capture both vertical and rotational displacement in connection with the *Phyphox* application or *Coach7* system.

Using *InsightMaker* and *Coach*, we investigated the time evolution of the energy associated with the vertical and torsional oscillation motions, as well as the coupling energy and the dissipated energy, to verify the compatibility of the model with the principle of energy conservation

References

[1] R. L. Wilberforce, "On Vibrations of Loaded Spiral Spring", *Phil. Mag*. **38** (1895) 386

[2] G.Torzo, M.Zanichelli, S.Pasqualotto, M.D'Anna "The Wilberforce pendulum: a complete analysis using micro accelerometer/gyroscope, smartphone and dynamical modeling*" Lat. Am. J. Phys. Educ*. Vol. 19, No. 4, March 2025

[3] http://insightmaker.com

[4] https://cma-science.nl/coach7_desktop_downloads_en